

SYLLABUS

Mathematics for Political Science

Political Science 598

Fall 2017

Mo/We 10:00-11:30am

7603 Haven Hall

Instructor: Iain Osgood
6741 Haven Hall
iosgood@umich.edu
Office hours: Tu: 2-5.

GSI: Patrick Wu
6564 Haven Hall
pywu@umich.edu
Hours: Tu:11:00-12:00; Th: 3-5.

This course provides an introduction to a set of mathematical concepts and skills commonly used in formal and empirical methods in political science. These include: multivariate calculus techniques and applications; certain fundamental topics in the study of probability and distributions; common techniques in linear algebra; and a detailed examination of optimization. While a greater weight will be placed on preparing students for coursework on formal theory, most of the concepts examined are regularly employed in the study of empirical methods, and lectures and problem sets will use examples from both formal and empirical methods.

Three objectives will be pursued throughout the course. First, to develop fluency with all of the concepts and techniques described in detail in the biweekly meetings laid out below. Second, to increase comfort with the symbolic rendering of real world concepts, formal reasoning, and methods of proof. Third, to improve understanding of mathematics in both formal and quantitative studies in political science. In other words, to make following research which employs mathematics easier, and to provide you with a set of commonly used skills which will help in developing your own research.

Readings: There are no required books for this course. The main concepts will be covered in lecture notes with definitions and examples. However, I do recommend the following books (and provide relevant chapters and pages) if you would like extra coverage of the topics from the course.

Simon, Carl P., and Lawrence Blume. *Mathematics for Economists*. Vol. 7. New York: Norton, 1994.

Sundaram, Rangarajan K. *A First Course in Optimization Theory*. Cambridge: Cambridge University Press, 1996.

These texts will be abbreviated in the reading list using the abbreviations *SimBlu* and *Sun*, respectively.

Course requirements: The course requirements comprise the following components.

Lecture and section participation: An important component of this course is active engagement with the material in both lecture and section. This also means benefitting from interacting with the instructors and fellow students. For this reason, attendance at all lectures and sections is mandatory, with the obvious exceptions of illness, personal issues, religious holidays or family emergency. If you miss more than six course meetings, I will deduct 20% from your final grade.

Problem sets: The course will have 8 problem sets, all of which must be submitted one week after they are released. The purpose of problem sets is to learn key concepts and their application through experience and repeated interaction. You should feel free to consult with your classmates (and the instructors in office hours) but all final submitted answers should be fully understood and be your own work product. Problems sets will determine 50% of the course grade.

Midterm examination: There will be a midterm examination held in class on Wednesday, October 25th. The midterm will cover a select set of the most important concepts discussed in the course up to that point. The emphasis will be on understanding and usage of the concepts, so you should focus your study on understanding the concepts and working through *brief* and memorable examples which illustrate how to employ particular techniques or ideas. The midterm will count towards 20% of the course grade.

Final examination: The final exam will be held on Monday, December 18th from 1:30pm-3:30pm. I will not reschedule the exam for any reason, barring illness or other emergency. It will cover all of the material from the course, but with an emphasis on concepts covered after the midterm examination. The final exam will determine 30% of the course grade.

Office hours: My office hours will be held in 6741 Haven Hall on Tuesdays from 2:00-5:00pm. If you cannot meet at this time, please send me an email and we will arrange a separate time to meet up.

Accommodations for Students with Disabilities: If you think you need an accommodation for a disability, please let me know as soon as possible. Some aspects of this course may be modified to facilitate your participation and progress. We can work with the Office of Services for Students with Disabilities to determine appropriate academic accommodations. SSD (tel.: 734-763-3000) typically recommends accommodations through a Verified Individualized Services and Accommodations (VISA) form. Any information you provide is private and confidential and will be treated as such.

Class topics and readings

I: Topics in Calculus and Probability

The expectation operator

sample space · random variable · PMFs/PDFs/CDFs/distributions · expected value of a random variable

· integration by parts · linearity of expectation · expectations of functions of random variables · variance · properties of the variance operator

No assigned readings.

Multivariate calculus: Multiple integrals

multidimensional integrals · Fubini's theorem · integrals with separable functions · multidimensional integrals with polar coordinates · the Gaussian integral

Paul's Online Math Notes: Double Integrals; Iterated integrals; and, Double integrals over general regions.

Joint distributions and further issues with the expectation operator

joint distributions of random variables · marginal distribution · conditional distribution · conditional expectation · independent random variables · covariance · correlation · variance of sums of random variables · variance-covariance matrix · multivariate normal distribution

No assigned readings.

Multivariate calculus: partial derivatives, derivatives and the Hessian

multivariate functions · vector-valued functions · partial derivative · the derivative and gradient vectors · second own partial derivative · second cross partial derivative · Hessian matrix · continuous differentiability

SimBlu 13.1; 14.1-14.3. 14.8.

Multivariate calculus: total derivatives, directional derivatives, and differentiating along a curve

total derivative · 'direct' and 'indirect' effects · level curves · directional derivatives · Euclidean distance · unit vectors · normalized directional derivatives · the second directional derivative · curves · the velocity vector · differentiating along a curve

SimBlu 14.4-14.6.

Taylor series expansion and other approximations

local linear approximation · the differential · the total differential · nonlinear approximations with the Taylor series · Taylor series expansion · kth order Taylor polynomials · multivariate Taylor series · Maclaurin series

SimBlu 2.7; 30.2-30.3.

II: Linear Algebra

The Algebra of Vectors

vectors in Euclidean space · vector operations · properties of vector addition and multiplication · length and direction of vectors · norm · p-norms · Euclidean distance metric · inner product · inner product and

angle (the cosine formula) · orthogonality · triangle inequality · Cauchy-Schwarz inequality

SimBlu 10.1-10.6.

Systems of Linear Equations I

linear equations · systems of linear equations · linear combinations · method of substitution · Gauss-Jordan elimination · augmented matrix · matrix row operations · (reduced) row echelon form · basic and free variables · rank · nonsingular matrices · linear independence · (non)homogeneous systems of linear equations · the zero solution · results on existence and number of solutions

SimBlu 7.1-7.4.

Systems of Linear Equations II

matrix inversion · singular/noninvertible · nonsingular/invertible · 2×2 inverse · determinants · minors/cofactors/adjoins · Laplace expansion · properties of inverses · properties of determinants · Cramer's rule

SimBlu 8.1-8.4; 9.1-9.2.

Vector spaces

Linear independence · spanning vectors · basis vectors · dimension of a space · vector spaces and subspaces · row space · column space · null space · the fundamental theorem of linear algebra

SimBlu 11.1-11.3; 27.1-27.5.

Eigendecomposition

eigenvectors · eigenvalues · characteristic equations/roots · eigendecomposition · diagonalization · defective matrices · repeated eigenvalues · spectral decomposition · matrix inversion using eigendecomposition · principal components analysis · systems of linear difference equations

SimBlu 23.1-23.3; 23.7.

Quadratic forms

monomials · quadratic forms · positive (negative) definiteness · positive (negative) semidefiniteness · indefiniteness · (leading) principal minors · tests for definiteness using leading principal minors · tests for definiteness using eigenvalues

SimBlu 13.3; 16.1-16.2.

In-class midterm examination

III: Prepare to Optimize

Sets and relations

sets and elements · cardinality · finite/infinite sets · (un)countable sets · set relations (equality, subset,

disjoint) · set operations (unions, intersections, differences, complements, products, power sets) · relations · domain/codomain · preimage/image · correspondence · function · surjective (onto) · injective (one-to-one) · bijective · function inverse · invertible

Sun 1.2.10.

Sequences

sequences · limit of a sequence · convergence/divergence · bounded sequences · accumulation point · limsup/liminf · Cauchy sequence · monotonicity · properties of sequential limits · Cauchy sequences · subsequence · the Bolzano-Weierstrass theorem

Sun 1.2.1-1.2.6.

Open sets, closed sets and compactness

Euclidean space · metric spaces · topological space · open ball · open sets · closed sets · unions and intersections of open and closed sets · compact sets · open covering · (finite) subcovering

Sun 1.2.7-1.2.8.

Function limits and continuity

functional limits · one-sided limits · continuity of a function ($\delta - \epsilon$ definition) · (the inverse) image of a function under a set · continuity (topological definition) · compositions and continuity · continuity preserves closed/compact/connected · intermediate value theorem · upper- and lowersemicontinuity

Sun 1.4.1-1.4.2.

IV: Optimization

Definition and Existence of Optima

upper and lower bounds of a set · supremum/infimum · maximum/minimum of a set · existence of a maximum/minimum of a set · optimization/maximization/minimization problem · feasible or constraint set · objective function · maximum/minimum of a function · solutions · maximizers/minimizers · the Weierstrass theorem

Unconstrained Optimization

interior points · local/global maxima/minima · unconstrained local maxima/minima · first order conditions (FOCs) · critical points · second order conditions · saddle points

Sun 4.1-4.5.

Convexity and Quasiconvexity

convex sets · epigraph/subgraph · convex/concave functions · tests for concavity · implications of concavity (for unconstrained optimization) · quasiconvex/concave functions · upper/lower contour sets · tests for quasi-concavity · implications of quasiconcavity (for unconstrained optimization)

Sun 7.1-7.5. Sun 8.1-8.5.

Constrained Optimization I: Equality Constraints and the Lagrangian

equality constraints · the Lagrangian · Lagrange multipliers · constraint requirement · checking for maxima/minima · the bordered Hessian

Sun 5.1-5.5

Constrained Optimization II: Inequality Constraints

inequality constraints · the Lagrangian with inequality constraints · binding/slack constraints · Kuhn-Tucker conditions · complimentary slackness conditions · the bordered Hessian with inequality constraints

Sun 6.1-6.3.

Comparative Statics Analysis

endogenous variables (and choice variables) · exogenous variables (parameters) · comparative statics analysis · elasticities · solution function · value function (and indirect utility function) · the envelope theorem · the envelope theorem with constraints

No assigned reading.

Comparative Statics and Implicit Functions

comparative statics · explicit functions · implicit functions · the implicit function theorem with one endogenous variable · the implicit function theorem with multiple endogenous variables

SimBlu 15.1-15.2.

Properties of Correspondences

upperhemicontinuity · lowerhemicontinuity · continuity of a correspondence · closed-/compact-/convex-valued correspondences · graph of a correspondence · closed graph

Sun 9.2

The Theorem of the Maximum

maximum theorem under strict convexity · maximum theorem

Sun 9.2-9.3.

Fixed point theorems

Nash equilibrium · fixed point of a function · Brouwer's fixed point theorem · fixed point of a correspondence · Kakutani's fixed point theorem

Sun 9.4.

Finite horizon dynamic programming

state and choice variables · transition and reward functions · feasible action correspondence · finite horizon dynamic programming problem · principle of optimality · Bellman equation · backwards induction